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Vector Diagram Correlation Of Leading And Lagging Power Factor In Salient Pole Synchronous Machine

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Abstract: - The paper highlights vector diagram construction of salient pole synchronous machine (motor and generator) both at a leading and at a lagging power factor regimes. The complexity of the vector diagram construction is more pronounced in the leading angle power factor than in the lagging mode. The presented problem is the difficulty involved in vector diagram construction mostly during courses and in practical life. The empirical analysis associated with this is obvious in contrast with the ruler and compass work in the real-time mode of construction. Real values of synchronous motor design were taken from a dissertation to explain in details the processes involved in the construction of the two common vector diagrams of salient pole synchronous machine at both leading and lagging power factor operations respectively. However, the construction of vector diagram of a salient pole synchronous generator goes beyond leading power factor regime to vector diagram with specific interest-based mode of operation. The result of the work is that the real processes taking place in the machine operation usually on load will be illustrated diagrammatically. The importance of vector diagram of synchronous machine is that; it helps the engineer to define at a glance the mode and operating characteristics of the machine without undergoing the rigour of having to read through the numerous equations associated with the characteristic mode of operation of salient pole synchronous machine.

Keywords: - *excitation voltage, lagging, leading, power factor, salient pole, synchronous machine*

I. INTRODUCTION

The easiest way to access the operating characteristics of a synchronous machine is to study its vector diagram. The vector diagram quickly defines the mode of operation of a machine – whether it is operating on leading angle or lagging angle power factor. The paper is examining two types of vector diagrams and both of them are illustrated in the work with full explanation on the procedures for their construction. The parameters used in the constructed diagrams are real values taken from a dissertation to illustrate the two common types of vector diagrams of a synchronous machine. The constructed diagrams look so divergent and dissimilar, yet they have certain things in common, namely: the power factor $\cos\varphi$, the load emf, the stator voltage and current, the resultant emf E_r with its associated angle and other common characteristic losses. Vector diagram is the result of empirical calculation in diagrammatic form. The leading features in the construction of vector diagram are the stator voltage, the current and the power factor angle. With these values at hand, the real construction is done using ruler and compass at great precision. The first thing is to state whether the angle is leading or lagging in order to define the position of the current or voltage. Leading power factor vector diagrams are drawn using supplementary angles while lagging power factor angles are drawn using complimentary angles. Leading power factor means that the current is leading the voltage by the giving power factor angle and lagging power factor means that the current is behind the voltage by the given power factor angle. In essence, the real aim of drawing the vector diagram of a synchronous motor is not just to show the position of voltages and currents. It is more than that.

The aim of constructing vector diagram of a machine is to define the proportionality of operating parameters both in magnitude and direction. There are the secondary features, apart from voltage and current, why it is necessary to draw the vector diagram of machines so that these features can be represented and studied. This is the shortest and concise way of conducting performance evaluation of synchronous machines apart from the programming method. From the representation made on the two distinctive vector diagrams, it can easily be said that the vector diagram of a lagging power factor will always lie on the first quadrant while the vector diagram of a leading power factor will always lie on the second quadrant. One distinguishing feature of a leading angle synchronous machine over the other types of machines is that during operation, with load, the stator input voltage is raised by the addition of two different emfs, the field and collinear phasor which also raise the stator input voltage. For synchronous electric motors, it will be convenient to distinguish vector diagrams as having leading and lagging power factor, but for generators, it may be of interest to draw vector diagram to highlight a specific operating condition of the generator.

Salient pole synchronous generator has a wide range of vector diagram construction, namely: (i) Vector diagram at active-capacitive load, (ii) at active-inductive load, (iii) at unsaturated regime, (iv) at short circuit regime, (v) at cosφ = 0, (vi) at cosφ = 1, (vii) with component voltages, (viii) with series impedance, (ix) at lagging power factor, (x) at leading power factor, (xi) at R ≠ 0, (l) at R = 0, (xiii) at an isolated generator with load, (xiv) with d-q armature reactances, (xv) with d-q synchronous reactances, (xvi) with collinear phasor. Of all these samples of salient pole synchronous generator vector diagrams, only ix, xii, xiv and xvi will be discussed here.

II. DATA FOR PARAMETER CALCULATION OF SALIENT POLE SYNCHRONOUS MOTOR

Line voltage: $V_L = 415 Volts$
Stator current per phase: $I_a = 371.86\, A$
Stator copper resistance: $R_s = 0.1\, ohm$
Stator d-axis synchronous reactance: $X_d = j0.4478\, ohm$
Stator q-axis synchronous reactance: $X_q = j0.3187\, ohm$
$Armature\ reactance$: $jX_1 = \sqrt{(X_d + X_q)^2} = j0.5496\, ohm$
$Stator\ voltage\ loss$: $I_a R_s = 371.86 * 0.1 = 37.2\, Volts$
Stator d-axis voltage: $E_d = jI_d X_d = j161.83 * 0.4478 = j72.47\, V$
Stator q-axis voltage: $E_q = jI_q X_q = 334.67 * 0.3187 = j106.66\, V$
Speed n = 1000 rpm, Cosφ = 0.9 and p = 6.
These data were taken from a dissertation to illustrate vector diagram construction [1].

III. VOLTAGE AND CURRENT EQUATIONS [2]

Load emf: $E_L = V_L + I_a * R_s = 415 + (371.86 * 0.1) = 452.2\, Volts)$ …………………………………(1)
Resultant emf: $E_r = V_{an} - I_a * R_s - jI_a * X_q - jI_d * X_d - jI_q * X_q$ ……………………………………...(2)
$239.6 - 371.86 * 0.1 - j334.67 * 0.3187 - j371.86 * 0.3187 - j161.83 * 0.4478 = 239.6 - 37.2 - j118.51 - j72.47 - j106.66 = 202.4 - j179 - j118.51 = \sqrt{3(202.4 - j179)} - j118.51 = 350.57 - j310.04 - j118.51 = 350.57 - j428.55 = 553.67\, Volts \angle 50.72°$, where $V_{an} = \frac{V_L}{\sqrt{3}}$ ………………….(3)
Armature reaction voltage
$E_a = I_a R_s + jI_a X_1 = 371.86 * 0.1 + j371.86 * 0.5496 = 37.2 + j204.37 = 207.73\angle 79.68°$ ……......(4)
Stator direct and quadrature axis currents: $I_a = I_q + I_d = I_a cos\varphi + I_a sin\varphi = 371.86 cos\varphi(154.2°) + j371.86 \sin(154.2°) = 371.86 * 0.9 + j371.86 * 0.4352 = 334.67 + j161.83\, A$ ……………………..(5)
Field emf: $E_f = V_{an} - I_a * R_s - jI_d * X_d - jI_q * X_q = 239.6 - 371.86 * 0.1 - j161.83 * 0.4478 - j334.67 * 0.3187 = 202.4 - j72.47 - j106.66 = 202.4 - j179 = 270\, Volts \angle -41.49°$ ……………………...(6)
Q-axis armature synchronous reactance voltage:
$I_a * X_q = 371.86 * j0.3187 = 118.51\, Volts \angle 90°$ ……………………..(7)
Collinear phasor: $E_f^{'} = V_{an} - I_a(R_s + jI_q) = 239.6 - 371.86 * 0.1 - j371.86 * 0.3187 = 202.4 - j118.51 = 234.54 \angle -30.35°$ ………………………………………………………...(8)
Adding the two vectors of the field emf and the collinear phasor gives
$E_f - E_f^{'} = 202.4 - j179 - (202.4 - j118.5) = -j60.50\, Volts$ …………(9)
$E_f + E_f^{'} = 202.4 - j179 + 202.4 - j118.51 = 404.8 - j297.51 = 502.36 \angle -36.31^{o}$

IV. PLOTTING SCALE

The scale used in the plotting of the vector diagram is: m = 5Volts/mm and m = 5 amperes/mm respectively.
Length of voltage vector $V_1 = \frac{V_L}{m} = \frac{415}{5} = 83mm$
Load emf: $E_L = \frac{452.2}{5} = 90.44mm$
Vector of stator current: $I_a = \frac{371.86}{5} = 74.372\, mm$
$E_a = \frac{207.73}{5} = 41.55mm$
Vector of q-axis stator current: $I_q = \frac{j334.7}{5} = 66.94 \angle 90°\, mm$
Vector of d-axis stator current: $I_d = \frac{j161.83}{5} = 32.4\angle 90°\, mm$
Vector of q-axis stator voltage loss: $E_q = I_q jX_q = 334.67 * \frac{j0.3187}{5} = 21.33 \angle 90°\, mm$
Vector of d-axis stator voltage loss: $E_d = I_d jX_d = 161.83 * \frac{j0.4478}{5} = 14.50 \angle 90°\, mm$

V. CONSTRUCTION OF VECTOR DIAGRAM OF A LAGGING POWER FACTOR MOTOR

The vector diagram of a lagging power factor motor will be constructed using the scale and parameters as in section 4.0. First, the stator current is located on the first quadrant at 0^0 and the voltage magnitude is located at a point 25.84^0 from the stator current at x-coordinate as shown in Fig. 1. The stator active voltage loss:

$I_a * R_s = 371.86 * 0.1 = \frac{37.2}{5} = 7.44mm$ ……………………………………………….(10)

The armature reactance voltage: $I_a * jX_1 = 371.86 * 0.5496 = \frac{204.37}{5} = 40.87mm$ ……(11)

In Fig. 1, V_L is located on the diagram with the length, $V_L = \frac{415}{5} = 83mm\ at\ 25.84^0$ from the vector of stator current. At the tip of V_L, the stator active voltage loss $I_a * R_s$ is located with the length 7.44mm. The vector $I_a * R_s$ is parallel to I_a (the stator current) and in opposite direction to each other and also out of phase with the stator voltage as shown in Fig. 1. The voltage vector $I_d * jX_d$ and $I_q * jX_q$ are divergent from each other at right angles [3]. The armature leakage voltage jI_aX_l with length 40.87mm is located at right angles with the stator active voltage loss. The vector $\overline{OC}$ forms the load voltage E_L. The d-axis voltage jI_dX_d by calculation is 72.5 Volts or 14.5mm. This voltage is located at the apex of jI_aX_l voltage to form right angles with the q-axis voltage jI_qX_q=106.66Volts or 21.33mm. The vector $\overline{OQ}$ is joined parallel to $\overline{ZQ}$ which is the reactance emf of the quadrature axis of the synchronous motor to form the resultant voltage E_r.

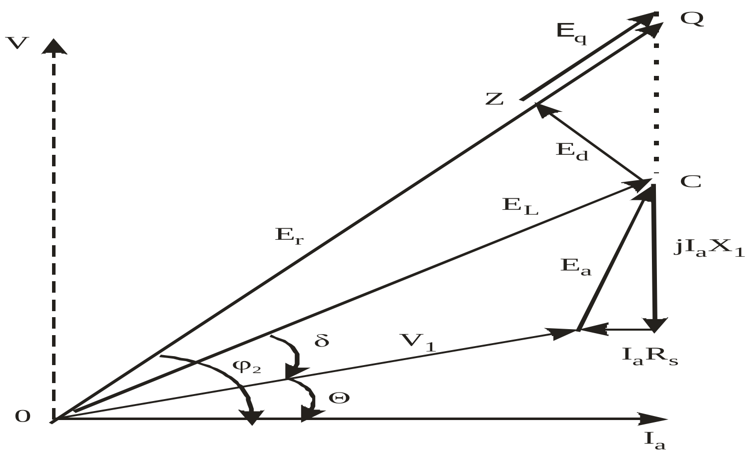

Fig.1 Vector diagram of a lagging power factor angle

From the diagram; $\theta = 25.84°$, $\varphi = 41.84°$, $\delta_r = 12.5°$, $\varphi_2 = 30°$ and $\delta = 16.5°$

Generally, in both leading and lagging power factor salient pole synchronous motor, the magnitude of q-axis current vector is greater than d-axis current vector. At a lagging power factor regime (reduced or complete absence) of excitation current, more current than voltage is being drawn from the power supply by the motor [4]. This makes lagging power factor synchronous motor to operate partly as an induction motor at full load and this is the reason for the increased load emf as high as 575 volts by measurement from Fig. 1, equivalent to $\sqrt{2} * V_L$. It is important to state here that all synchronous motors do start as induction motors but run as pure synchronous motors at rated excitation voltage. The ability of lagging power factor motors to generate reactive emf makes them even more applicable in power factor correction by introducing negative VAR into the system [5].

VI. DATA FOR PARAMETER CALCULATION OF SALIENT POLE SYNCHRONOUS MACHINE [6]

Leakage inductance: X_l = 0.1235 ohm
Mutual inductance of the d-axis stator winding: $X_{\Phi d}$ = 0.4067 ohm
Mutual inductance of the q-axis stator winding: $X_{\Phi q}$ = 0.2773 ohm
D-axis synchronous reactance of the stator winding: $X_d = X_l + X_{\Phi d}$ = 0.1235+0.4067 = 0.5302 ohm ……(12)
Q-axis synchronous reactance of the stator winding: $X_q = X_l + X_{\Phi q}$ = 0.1235+0.2773 = 0.4008 ohm ……..(13)
Mutual magnetizing reactance: $X_\Phi = X_{\Phi d} + X_{\Phi q}$ = 0.4067+0.2773 = 0.684 ………………………………...(14)
Synchronous reactance of the stator winding: $X_s = X_\Phi + X_l$ = 0.684+0.1235 = 0.8075 …………………….(15)
Armature winding leaking voltage: $j\bar{I}_aX_l$ = j371.86*0.1235 = j45.92Volts

Armature winding magnetizing voltage E_A: $j\bar{I}_a X_\Phi$ = j371.86*0.684 = j254.35Volts
$Amature\ winding\ leakage\ voltage$: $jI_a X_l = j371.86 * 0.1235 = j45.92\ Volts$
Q-axis mutual voltage: $E_{\phi q} = jI_q X_{\phi q} = 334.67 * j0.2773 = j92.8\ V$
Mutual voltage of the d-q axes: $j\bar{I}_a(X_d - X_q)$ = j371.86*(0.5302-0.4008)=j48.12 Volts ……………………..(16)
Field voltage: $\bar{E}_f = \bar{V}_{an} + R_s\bar{I}_d + R_s\bar{I}_q + jX_d\bar{I}_d + jX_q\bar{I}_q = 239.6 + 0.1 * 161.83 + 0.1 * 334.67 + j0.5302 * 161.83 + j0.4008 * 334.67 = 239.6 + 16.183 + 33.467 + j85.80 + j134.14 = 289.25 + j219.94 = 363.372\ \angle 37.25°\ Volts$ ……………………………(17)
The field collinear phasor: $\bar{E}_f^{\prime} = \bar{V}_{an} + \bar{I}_a(R_s + jX_q) = 239.6 + 371.86(0.1 + j0.4008) = 276.786 + j149.04 = 314.363\ \angle 28.3°\ Volts$ …………………(18)
Resultant voltage: $\bar{E}_r = \bar{V}_{an} + R_s\bar{I}_a + jX_q\bar{I}_a + jX_d\bar{I}_d + jX_q\bar{I}_q = 239.6 + 371.86 * 0.1 + j371.86 * j0.4008 + j161.83*0.5302 + j334.67*0.4008 = 276.786 + j149.04 + j85.8 + 134.14 = 276.786 + j368.98 = 461.26\ \angle 53.16°\ Volts$ ……………………………..(19)
Addition of field and collinear voltages: $\bar{E}_f + \bar{E}_f^{\prime} = 289.25 + j219.94 + 276.786 + j149.04 = 556.04 + j368.98 = 676.68\ \angle 33.1°\ Volts$ …………………...(20)
Subtraction of collinear phasor from field voltage: $\bar{E}_f - \bar{E}_f^{\prime} = 289.25 + j219.94 - 276.786 - j149.04 = 12.464 - j70.9\ Volts$ ………………………………..(21)

VII. PLOTTING SCALE

The plotting scale is taken as shown below, thereby given the following vectors
$j\bar{I}_a X_l = \frac{j45.925}{5} = 9.2mm$; $\bar{E}_f^{\prime} = \frac{314.363}{5} = 62.87mm$; $j\bar{I}_a X_\Phi = \bar{E}_A = \frac{j254.35}{5} = 50.87$mm;
$j\bar{I}_a(X_d - X_q) = \frac{48.12}{5} = 9.63mm$; $\bar{E}_f = \frac{363.372}{5} = 72.674mm$; $\bar{E}_f^{\prime} = \frac{314.363}{5} = 62.87mm$;
$\bar{E}_r = \frac{461.26}{5} = 92.25mm$; $\bar{E}_f + \bar{E}_f^{\prime} = \frac{676.68}{5} = 135.34mm$

VIII. CONSTRUCTION OF VECTOR DIAGRAM OF A LEADING POWER FACTOR MOTOR

Using the compass with the origin at O, the stator current I_a is located with the leading angle of 154.2^0 or $(180-25.8^0)$ on the second quadrant at a length of 74.372mm. The line voltage is located anticlockwise using the angle between voltage and current, which is 25.8^0 leading at a length of 83mm. The quadrature and direct axes currents lay on the x-axis and the y-axis respectively at an angle of 90^0 from each other as shown in Fig.2. The field voltage is located from the fact that the q-axis voltage loss is parallel and in opposite direction with the q-axis current I_q and at right angle with the stator active voltage loss I_aR_s.

The stator quadrature axis voltage loss jI_aX_q meets the stator active voltage loss I_aR_s at an angle 90^0 with the magnitude of j118.5Volts at a length of 23.7mm on the drawing scale. Having plotted the stator quadrature axis voltage loss and the active voltage loss with the corresponding lengths, a straight line is drawn from the end of I_aR_s to the origin O to form the field voltage with the length 72.67mm. By measurement using compass, the field voltage subtends an angle of 138.51^0 $(180^0 - 41.49^0)$ from the origin. It is seen by calculation that the angle δ between the line voltage and the field voltage (torque angle) is 8.29^0 equation 22) and 9^0 by measurement, the angle φ between the field emf and the stator current is 17.55^0 or φ = θ – δ [7], where θ = 25.84^0, the leading phase angle between the stator voltage and current. The scale of proportionality used in the vector diagram gives the parameter sizes as they would appear in realtime and can be used to analyze the characteristic operation of the synchronous motor.

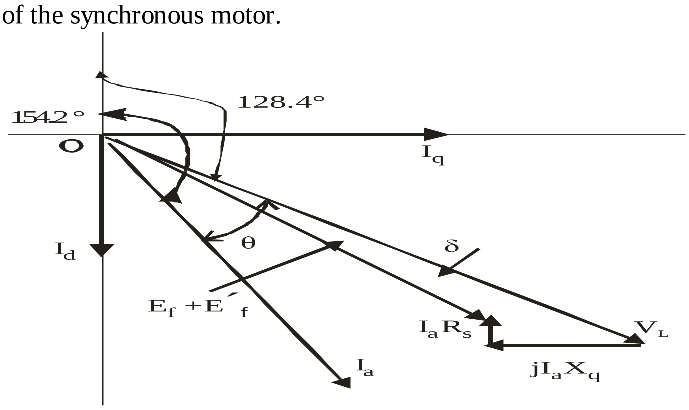

Fig. 2 Vector diagram of a leading power factor angle synchronous motor

From the diagram, $\delta = 9°$; $\theta = 25.84°$; δ_r = nil and $\varphi = 16.84°$.

Table 1. Results of vector diagram construction of electric motor

S/N	Unit	Lagging power factor motor	Leading power factor motor
1.	V_1	415 Volts	419.315 Volts
2.	E_{load}	575 Volts	452.2 Volts
3.	E_r	675 Volts	600 Volts
4.	θ	25.84^0	154.16^0
5.	φ_2	54.84^0	--
6.	δ_r	12.5^0	--
7.	δ	16.5^0	-8.29^0
8.	φ	41.84^0	-16.84^0
9.	$E_{\Phi q}$	j92.8 Volts	-j60.49 Volts

where, for leading power factor synchronous motors, stator input voltage
$V = V_L + (E_f - E_f^{'}) = 415 - j60.49 = 419.315\angle - 8.29^0$ …………..(22)

IX. CONSTRUCTION OF VECTOR DIAGRAM OF LAGGING POWER FACTOR SYNCHRONOUS GENERATOR

The vector diagram of a lagging power factor synchronous generator resembles the one drawn for a lagging power factor synchronous motor in many ways. Yet it is distinguishable. Truly speaking, the difference between a synchronous motor and a synchronous generator is only observed during their operation and not necessarily on their constructive design. And so the similarity in the VD presented in this paper is not by accident. For electric motor action, torque acting on the rotor shaft must be in the same direction of the angular speed ω_s to give a positive torque with the rotor. For this reason, the torque angle δ and the rotor angle δ_r must assume negative sign and should be greater or equal to -90°. For generator action, the reverse is the case here; the rotor shaft rotation goes opposite to the direction of torque for positive values of δ and δ_r less or equal to +90° [2].

In the construction of VD for lagging power factor motor, the same plotting scale was used to get what is shown in Fig. 3. The major parameters of the vector diagram – stator current X_a and voltage are located as shown with the subtended angle θ. The stator parameters $R_s\bar{I}_a$ *and* $j\bar{I}_aX_l$ determine the vector of the resultant voltage $\bar{E}_r$ corresponding to a calculated length of 92.25mm. The sum of the field voltage and the collinear phasor was located on the diagram at an angle of δ_1 and length of 135.34mm. The length of $j\bar{I}_aX_\Phi$ continued from the peak of $j\bar{I}_aX_l$ and terminated at point B in the diagram and was rounded up with the dashed line to meet the calculated sum of the field and collinear phasor. Seeing that line $\overline{AB}$ represented nothing in technical terms, the field and collinear phasor had to be relocated to meet the peak of $j\bar{I}_aX_\Phi$ vector at point B thereby completing the vector diagram. The scenario observed in the vector diagram of a lagging power factor generator shows that for such a generator, there is limit to the excitation voltage to maintain steady state operation. The adjustment made in the diagram now reduced the excitation voltage from $676.68\angle 33.1°$ $Volts$ to $600\angle 28.5°$ $Volts$. In the diagram, torque angle $\delta = 28.5°$, rotor angle $\delta_r = 22°$ and the angle subtended by phase voltage and resultant voltage is $\alpha = 6°$.

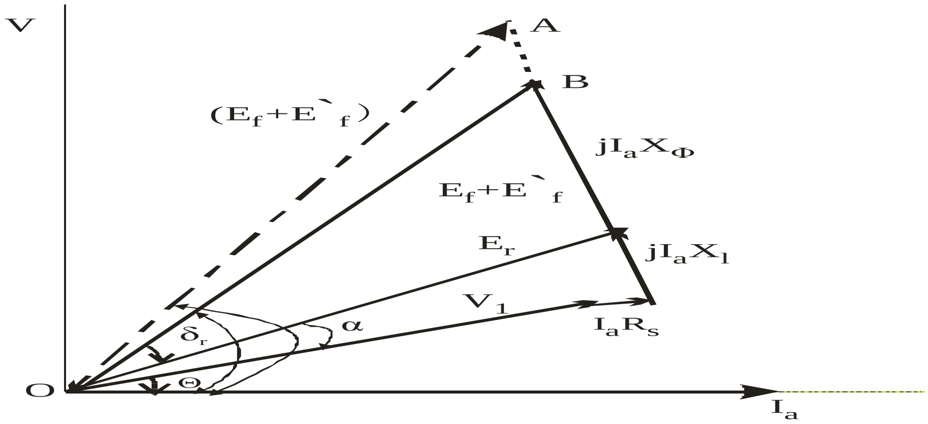

Fig. 3. Vector diagram of a lagging power factor synchronous generator

From the diagram, $\varphi_1 = 62.1°$, $\varphi = 53.84°$, $\delta_r = 22°$, $\delta = 28.5°$; $\theta = 25.84°$ and $\alpha = 6°$

X. CONSTRUCTION OF VECTOR DIAGRAM OF COLLINEAR PHASOR OF SYNCHRONOUS GENERATOR

The construction of vector diagram defining collinear phasor is done by first marking the position of the stator current at a leading angle of 154.16°. Then the line voltage V_1 is located with reference to the stator current at an angle $\theta = 25.84°$ as shown in Fig. 4. From the continuation of the line voltage, the armature voltage $R_s\bar{I}_a$ is located with the length 7.44mm. This armature voltage forms right angle with the q-axis voltage $j\bar{I}_a X_q$ with length 29.81mm. Joining the vector of the q-axis voltage to the origin produces the sum of field winding voltage $\bar{E}_f + \bar{E}_f^{\prime}$. The interaction of the quadrature axis voltage $j\bar{I}_a X_q$ and the field voltage reduces the field voltage vector to something equivalent to stator mutual axis voltage of the d-q axes $j\bar{I}_a\left(X_d - X_q\right) = \frac{48.12}{5} = 9.62mm\ in\ length$. The torque angle δ is obtained by measurement to be 22°.

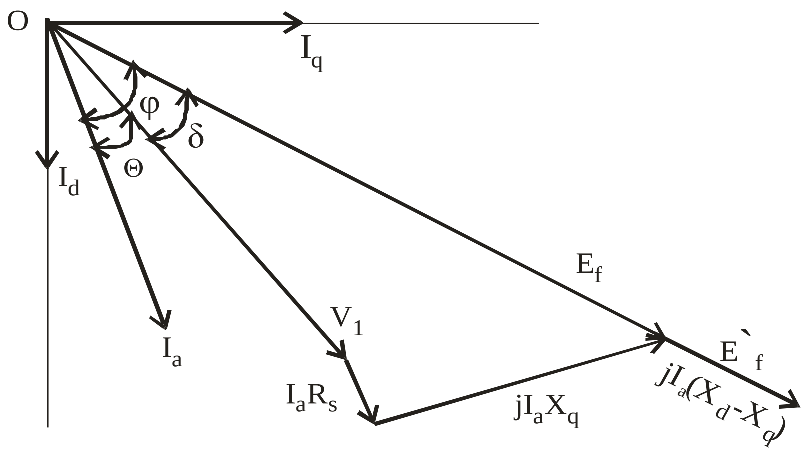

Fig. 4. Vector diagram of collinear phasor synchronous generator

From the diagram; $\varphi = 47.84°$, $\theta = 25.84°$, $\delta = 22°$ and δ_r = nil

XI. VECTOR DIAGRAM OF D-Q ARMATURE REACTANCES OF A LEADING POWER FACTOR GENERATOR

The vector diagram of d-q armature reactances of a leading power factor generator is concentrated on the quadrature axes values. It resembles the vector diagram of Fig. 3, only that it included resultant voltage $\bar{E}_r$. The stator current is the reference phasor. The stator voltage is located from the current using the leading power factor angle of 25.84° with a corresponding length of 83mm. The continuation of $\bar{V}_1$ is the armature leakage voltage $j\bar{I}_a X_l$ which terminated at the peak of the resultant voltage. The d-axis armature magnetizing voltage continues from the end of the resultant voltage to form a right angle with the q-axis armature magnetizing voltage $j\bar{I}_q X_{\Phi q}$. The peak of this vector is joined to the origin to form the field and collinear phasor which is measured to be 95.5mm or 487.5 Volts, by calculation this was 135.34mm or $676.68\angle 33.1°\ Volts$. The dashed line on the field voltage and the collinear phasor represents the limit of field voltage base on the machine parameters. The torque angle δ and the rotor angle δ_r were measured to be 20° and 13° respectively. The omission of armature active resistance in the vector diagram of Fig. 5 was intended to simplify the diagram. However, the assumed absence of active resistance of the armature winding is a theoretical intention to estimate the minimum torque of the generator according to the following expression [2, 8].

$3T_d = 3 * [(E_f E_r)/(\omega_s X_\Phi)] * sin\delta_r$ at $R \neq 0$ ……………………….(23)

$3T_d = (3E_f V_{an}/\omega_s X_s) * sin\delta_r$ at $R = 0$ ……………………………(24)

From equation (22) with particular reference to Fig. 5 gives:

$3T_d = 3 * \left[\frac{420*435}{104.72*0.684}\right] sin13 = 1721.32$ N-m, and for (23):

$3T_d = 3 * \left[\frac{420*239.6}{104.72*0.8075}\right] sin13 = 803.11$ N-m

where $\omega_s = \frac{2\omega}{p}$, $\omega = 2\pi f\ and\ \omega_s = \frac{4\pi f}{p} = 4\pi * \frac{50}{6} = 104.72rad.$ ………………..(25)

This shows that generator torque reduces with the reduction of stator active resistance R_s $eqn. 24$. Theoretically, when $R_s \ll X_s$, the vector diagram of a synchronous generator can be drawn neglecting active resistance. However, this is not obtainable practically because of temperature coefficient of winding conductors. It is certain that high torque can be favourable for electric motors but not for generators.

11.1 Common terms used in the work

θ = Angle between line voltage and stator current
φ = Angle between field voltage and stator current
δ_r = Angle between field voltage and resultant voltage (rotor angle)
α = Angle between resultant voltage and line voltage
δ = Angle between field voltage and line voltage (torque angle)

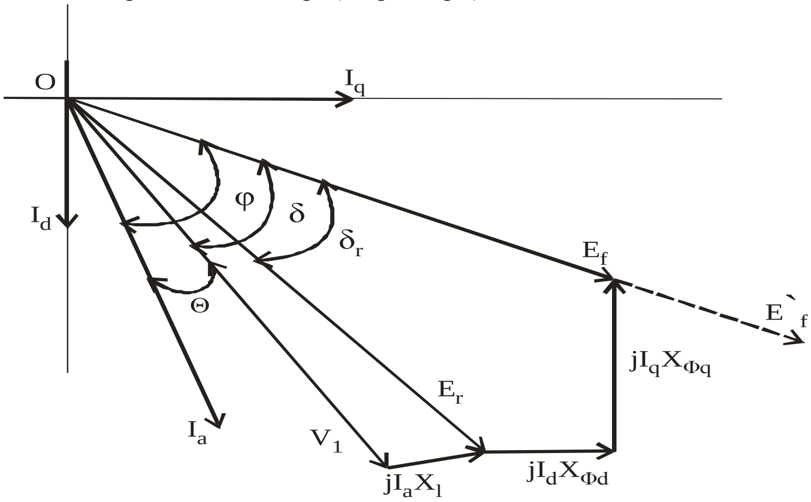

Fig .5. Vector diagram of d-q armature reactances of a synchronous generator

From the diagram; φ = 58.84°, θ = 25.84°, δ = 20° and δ_r = 13°

Table 2 Result of vector diagram construction of synchronous generator

S/N	Unit	Lagging power factor Fig. 3	Collinear phasor Fig. 4	d-q armature reactance Fig. 5
1.	V_1	415V	433.3V	433.3V
2.	E_r	475	--	435V
3.	$E_f + E_f^{\iota}$	515V	490V	420V
4.	φ	54°	47.84°	58.84°
5.	α	6°	--	7°
6.	δ	28.5°	22°	20°
7.	δ_r	22°	--	13°

where $V_1 = V_L + (E_f - E_f^{\iota}) = 415 + 12.464 - j70.9 = 433.3 \angle -9.42°$

XII. SUMMARY OF VECTOR DIAGRAM CONSTRUCTION

So far, it is evident that the aim of construction of vector diagram of salient pole synchronous machine is not just to reproduce the data and angles in diagrammatic form but it is also aimed at looking at the figures in diagrammatic form to see if there are areas that need adjustment to obtain optimum performance. Such was the case in almost all the vector diagrams considered in this paper. In Figure 1, the resultant emf E_r would have run to infinite value if not the d-axis emf which set the bound to it at point Z by superimposing the excess resultant voltage with the q-axis emf and by doing so limited E_r to a few units above the load voltage. In that capacity, the resultant emf is now acting as a compensator to the load voltage. In Fig. 2, the field and collinear phasor stand to be regulated by the q-axis voltage. During the vector diagram construction of figure 2, the major concern was to ensure that jI_aX_q meets I_aR_s at right angle. The fact that the field and collinear phasor will terminate at the apex of I_aR_s is not a fundamental issue here. The fundamental thing is that whatever comes out of the vector diagram will replace the empirically calculated result and will be used in further analysis of the motor. Figure 3 shows that the value of field and collinear phasor calculated using mathematical model could not have been plotted without difficulty. The dashed line shown in Fig. 3 was the calculated value of $E_f + E_f^{\iota}$ but this was however moderated by the magnetizing emf $jI_aX_\emptyset$. Because of this, $E_f + E_f^{\iota}$ had to be shifted from the calculated position at point A to the graphically moderated value at point B before a true picture of the machine operation was viewed in the diagram of figure 3. This action was necessitated by the fact that $\overline{AB}$ had no engineering representation in the diagram. Also, the fact that $I_a R_s$ must meet $jI_a X_l$ at right angle was taken into consideration. Figure 4 was moderated by the same principle as in Fig. 3 where all the field and collinear phasor were not put into use and had to be moderated by the quadrature axis voltage and the mutual axis voltage of the d-q axes $jI_a(X_d - X_q)$. Figure 5 best describes the vector diagram of a salient pole synchronous generator based on the d- and d-axes parameter representation and it fully expressed all the generator parameters. This is an example of a vector diagram illustrating zero stator active resistance. It is also the only vector diagram out of the five described here that had to be regulated by two right angles: firstly at the $V_L \leftrightarrow jI_a X_l$ vector and secondly at $jI_d X_{\emptyset d} \leftrightarrow jI_q X_{\emptyset q}$ vector. In almost all the vector diagrams shown in this paper, the main object of interest is the field voltage regulation or load voltage regulation as in Fig. 1. This shows that proper regulation of excitation voltage is very important for optimum performance of electrical machine. However, this regulating factor is a direct function of proper choice of impedances of the stator winding. Finally, the synchronism of electric machine is not necessarily achieved by increasing excitation voltage rather, it is a complex factor based on the right choice of active, inductive and passive parameters of the stator winding.

13. Results and Analysis

Comparatively, looking at all the vector diagrams, it is obvious that the vector diagram of a lagging power factor machine is plotted on the first quadrant while the vector diagram of a leading power factor machine is plotted on the second quadrant. Lagging power factor angles are complimentary 25.84^0 while leading power factor angles are supplementary 180^0 - 25.84^0. Lagging power factor implies that voltage is lagging or behind the current and therefore voltage takes its geometrical location on the position of the power factor angle of 25.84^0 with the x-coordinate as the reference point. On the other hand, leading power factor implies that voltage is leading the current and so the current is first positioned on the second quadrant at a coordinate location of 154.16^0. For lagging power factor angles, the reverse is the case. The current is located on 0^0 before locating the voltage at a coordinate of 25.84^0. Vector diagram helps to reveal some of the operating conditions of a machine which would not have been noticed with measuring instruments. Table 1 shows the parameters of a salient pole synchronous motor for leading and lagging power factor angles. The angle $cos\varphi_2$ is a function of the resultant emf and it is responsible for electromagnetic dispersion in magnetic cores of synchronous machines [1]. This angle is used during design of synchronous motor for the determination of q-d axes per unit armature reaction magnetomotive forces $F^{\prime}_{aq*}, F^{\prime}_{ad*}$, the maximum static overload (maximum moment) M_{max} and other related machine parameters.

$$\frac{F^{\prime}_{aq*}}{cos\varphi_2} = X_d * K_{ad} * F^{\prime}_{a,rated} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(26)$$

where $F^{\prime}_{a,rated}$ = ratio of stator mmf per pole to field winding mmf and the d-axis armature reaction mmf becomes:

$$F^{\prime}_{ad*} = X_d * K_{ad} * F^{\prime}_{a,rated} * sin\varphi_2 + K_h \tau cos\varphi_2/\delta. \quad \ldots\ldots\ldots\ldots..(27)$$

Maximum static overload: $M_{max} = \frac{E_{r*}}{\frac{F_{aq}}{cos\,\varphi_2}} * \frac{K_{p1}}{X_d * cos\varphi} \quad \ldots\ldots\ldots\ldots(28)$

In (27), δ represents airgap value while τ is the pole length. If this q-d axes armature reaction forces are reduced by additional load on the rotor shaft, the mmf may decay towards zero and the machine may fall out slip and come to a stop. Table 2 shows the parameters of a salient pole synchronous generator extracted from the vector diagrams. Figure 4 can also be called vector diagram of a leading power factor synchronous generator. The parameters of a synchronous generator can best be illustrated at a lagging power factor regime and at the vector diagram of d-q armature reactances regime. From the table, it is observed that once synchronous generator starts, the excitation windings induce additional emf into the stator windings resulting in the increase of V_1 as shown in the footnotes of the tables. This additional emfs is the difference between the excitation voltage and the collinear phasor mostly for leading power factor regimes. All synchronous motors operate on three levels of voltage – input voltage, field voltage or load voltage and resultant voltage. The two later ones are often ascertained through computation or vector diagram construction. The ability of synchronous motor to generate additional voltage higher than their input voltage is applied in reactive power compensation by adding positive var into the system.

XIII. CONCLUSION

In the vector diagram drawn for lagging power factor, the load voltage played important role in the study while in the leading power factor; the excitation voltage played the leading role in studying the operating characteristics of the synchronous generator. Also, in leading power factor, stator voltage loss depends on the field emf while in lagging power factor motor; stator voltage loss depends on the input line voltage V_1. The fact that quadrature axis current is greater than direct axis current is clear evidence that rotor speed is independent of load provided that the motor is operating at the maximum available torque. The fact that load voltage is greater than line input voltage makes synchronous motor to operate independent of load variation especially within the range of $\left(\frac{V_1-E_f}{V_1}\right) * 100\% = 3\%$ (third column) of Table 2 for leading power factor synchronous generator and $\left(\frac{E_L-V}{E_L}\right) * 100\% = 7.27\%$ for leading power factor motor, (second column) of Table 1.

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